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SPECIAL FUNCTIONS IN A DISCRETE LAPLACIAN

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It is known that the fundamental solution to

$$u_t(n,t) = \Delta_d u(n,t) := u(n+1,t) - 2u(n,t) + u(n-1,t), \quad n \in \mathbb{Z}, \quad t > 0,$$

with $u(n,0) = \delta_{nm}$ for every fixed $m \in \mathbb{Z}$, is given by $u(n,t) = e^{-2t} I_{n-m}(2t)$ (see [3] and [4]), where $I_k(t)$ is the Bessel function of imaginary argument. Consequently, the *heat semigroup* is given by the formal series

$$W_t f(n) = \Sigma_{m\in\mathbb{Z}} e^{-2t} I_{n-m}(2t) f(m).$$

This function is the solution to the *discrete* heat equation with initial data $\{f(n)\}_{n\in\mathbb{Z}}$. Other second order differential operators and the associated discrete heat kernels arise when dealing with equations connected with physics, see [4,5].

By using semigroup theory, the formula for $W_t f(n)$ will allow us to tackle problems in two different contexts.

- On one hand, we will define and analize some classical operators of the Harmonic Analysis associated with the discrete Laplacian Δ_d, such as maximal operators, square functions, and Riesz transforms [1].
- On the other hand we will be able to define the fractional powers of the discrete Laplacian on a mesh of size h and then to show the convergence to the fractional Laplacian on the whole space in the discrete supremum norm as $h \to 0$ [2].

Since the discrete heat semigroup is given in terms of modified Bessel functions, the careful and exhaustive use of some properties and facts about these functions is crucial to get the results.

The work is in collaboration with Ó. Ciaurri, T. A. Gillespie, P. R. Stinga, J. L. Torrea and J. L. Varona.

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