Separation of singularities, generation of algebras and complete K-spectral sets

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Abstract: In this talk, we will show a certain relation between the generation of uniform analytic algebras and complete K-spectral sets of Hilbert space operators.

Havin and Nersessian showed that, under certain geometric conditions on domains Ω_1, Ω_2 in $\mathbb{C}$, every function $f \in H^\infty(\Omega_1 \cap \Omega_2)$ can be written as $f = f_1 + f_2$, with $f_j \in H^\infty(\Omega_j)$. This result can be seen as a separation of singularities. Their result and techniques were used in our previous work to study the question of whether the collection of functions of the form $g \circ \varphi_j$, where $\{\varphi_1, \ldots, \varphi_n\}$ are fixed functions from Ω into $\mathbb{D}$, generates the algebra $H^\infty(\Omega)$ (or the algebra $A(\overline{\Omega})$).

After explaining these results, we apply them to studying complete K-spectral sets. Let T be an operator on a Hilbert space H. A compact subset X of $\mathbb{C}$ is said to be a complete K-spectral set for T if $\|f(T)\|_{\mathcal{B}(H \otimes \mathbb{C}^s)} \leq K \sup_{z \in X} \|f(z)\|_{\mathcal{B}(\mathbb{C}^s)}$, for every $s \times s$ rational matrix function f with poles outside of X of any size $s \geq 1$. Complete K-spectrality of an operator T in the closed unit disc $\overline{\mathbb{D}}$ is equivalent to the similarity of T to a contraction. An analogous result holds for any good simply connected domain, which involves the Riemann map $\overline{\mathbb{D}} \to \overline{\Omega}$. We will use our results on algebra generation to give tests for complete K-spectrality. These will have the form: "if $\|\varphi_k(T)\| \leq 1$ for every k, then $\overline{\Omega}$ is a complete K-spectral set for T, for some K." We generalize previous theorems of Badea, Beckermann, Crouzeix, B. Delyon, F. Delyon, Kazas, Kelley, Mascioni, Putinar, Sandberg, and others.

We generalize a result of Delyon and Delyon that says that every convex set containing the numerical range of an operator is a complete K-spectral set for this operator. We show how to apply this last result to obtain new criteria for similarity to a normal operator.

This is joint work with Dmitry Yakubovich (Univ. Autónoma de Madrid) and partially joint work with Michael Dritschel (Newcastle Univ.).