A Hankel matrix acting on conformally invariant spaces

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ABSTRACT: If μ is a finite positive Borel measure on $[0,1)$ and $n = 0,1,2,\dots$, we let μ_n denote the moment of order n of μ, that is,

$$\mu_n = \int_{[0,1)} t^n \, d\mu(t),$$

and we let $\mathcal{H}_\mu$ to be the Hankel matrix $(\mu_{n,k})_{n,k\geq 0}$ with entries $\mu_{n,k} = \mu_{n+k}$. The matrix $\mathcal{H}_\mu$ can be viewed as an operator on spaces of analytic functions in the unit disc $\mathbb{D}$ by its action on the Taylor coefficients:

$$a_n \mapsto \sum_{k=0}^{\infty} \mu_{n,k} a_k, \quad n = 0,1,2,\cdots.$$

To be precise, if $f(z) = \sum_{k=0}^{\infty} a_k z^k \in \mathcal{H}ol(\mathbb{D})$ we define

$$\mathcal{H}_\mu(f)(z) = \sum_{n=0}^{\infty} \left(\sum_{k=0}^{\infty} \mu_{n,k} a_k \right) z^n,$$

whenever the right hand side makes sense and defines an analytic function in $\mathbb{D}$. If μ is the Lebesgue measure on $[0,1)$ the matrix $\mathcal{H}_\mu$ reduces to the Hilbert matrix $\mathcal{H} = \left((n+k+1)^{-1}\right)_{n,k\geq 0}$, which induces the classical Hilbert operator.

The action of $\mathcal{H}_\mu$ on Hardy spaces has been studied in [1] and [2]. In this work we shall study the action of $\mathcal{H}_\mu$ on conformally invariant spaces of analytic functions in the disc. Among other results we shall characterize the measures μ for which $\mathcal{H}_\mu$ is well defined and bounded on the Bloch space and on $BMOA$.

References

[1] Ch. Chatzifountas, D. Girela and J. A. Peláez *A generalized Hilbert matrix acting on Hardy spaces*, J. Math. Anal. Appl. **413**, n. 1, (2014), 154–168.

[2] P. Galanopoulos, J. A. Peláez *Hankel matrices on Hardy and Bergman spaces*, Studia Mathematica, **200**, n. 3, (2010), 201–220.