On a nonlinear mean value property related to the p-laplacian

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ABSTRACT: Given a domain $\Omega \subset \mathbb{R}^n$ and a harmonic function $u : \Omega \to \mathbb{R}$, it is well-known that u satisfies the mean value property, that is, $u(x)$ coincides with the average of u over any ball $B_x = B(x, r(x)) \subset \Omega$ for each $x \in \Omega$. We consider a new (nonlinear) mean value property:

$$u(x) = \alpha \left(\frac{1}{2} \inf_{B_x} u + \frac{1}{2} \sup_{B_x} u \right) + \frac{1-\alpha}{|B_x|} \int_{B_x} u(y)dy,$$

where $0 \leq \alpha \leq 1$ is a constant. It turns out that this mean value property is related to the ∞-laplacian if $\alpha = 1$ and to the p-laplacian if $0 \leq \alpha < 1$. For $u \in C(\overline{\Omega})$, we define the operator $T_\alpha u(x)$ as the right-hand side of the previous equation.

We show that the Dirichlet Problem

$$\begin{cases} T_\alpha u = u & \text{in } \Omega \\ u = f & \text{on } \partial\Omega \end{cases}$$

has a unique solution $u \in C(\overline{\Omega})$, where $f \in C(\partial\Omega)$ is any given boundary data, under certain assumptions on Ω and $r(x)$. (Joint work with José G. Llorente, Universitat Autònoma de Barcelona).