Open problem:
Asymptotic of the Christoffel function in 2-variables

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Koornwinder's method to generate bivariate OP

- Let ω_1, ω_2 weight functions in the real intervals (a, b) and (c, d), respectively.
- Let σ a positive function in (a, b), such that:
  i) σ is a polynomial of degree 1,
  ii) or σ is the square root of a nonnegative polynomial of degree 2 and ω_2 is an even weight function in a symmetric interval $(-c, c)$.
- $p_{n,m}$ the n-th orthonormal polynomial with respect to the weight $\sigma(x)^{2m+1}\omega_1(x)$
- q_m, the m-th orthonormal polynomial with respect to ω_2,

Koornwinder's method to generate multivariate OP

Theorem (Koornwinder)

The polynomials

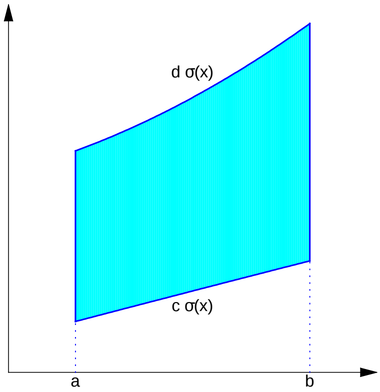
$$P_k^n(x,y) = p_{n-k,k}(x)\,\sigma(x)^k\,q_k\left(\frac{y}{\sigma(x)}\right),\ 0 \le k \le n,$$

are orthonormal with respect to the weight function

$$W(x,y) = \omega_1(x)\,\omega_2\left(\frac{y}{\sigma(x)}\right),\ (x,y) \in R,$$

where $R = \{(x,y) : a < x < b,\ c\sigma(x) < y < d\sigma(x)\}$.

Koornwinder's method to generate multivariate OP



Koornwinder's method to generate multivariate OP

We can use this method to construct OP with respect to some weight function on

- the square ($\sigma(x) = 1$)
- the unit circle ($\sigma(x) = \sqrt{1 - x^2}$)
- the simplex ($\sigma(x) = 1 - x$)
- a parabolic region ($\sigma(x) = \sqrt{x}$)
- $\ldots$

The reproducing kernels satisfy

$$K_n(W; x, y; x, y) = \sum_{k=0}^{n} K_{n-k,k}(x, x)\, \sigma(x)^{2k}\, q_k^2 \left(\frac{y}{\sigma(x)} \right)$$

where $K_{n-k,k}(x, u)$ denotes the reproducing kernel with respect to the weight $\sigma(x)^{2k+1}\omega_1(x)$

Remark

The reproducing kernel and the Christoffel function are independent of the particular choice of the family of orthonormal polynomials.

Open problem

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Can we deduce the asymptotic for reproducing kernel $K_n(W; x, y; x, y)$ (and the Christoffel function) from the asymptotic for the kernels associated to ω_1 and ω_2 ?