Uniform convergence of Hermite-Padé approximants for different systems of Markov type functions.

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Outline

1. Basics and recents results about convergence of Hermite-Padé approximants

2. Uniform convergence of Mixed type Hermite-Padé approximants

3. Future works

Markov's functions

Let s be a finite Borel measure with constant sign whose compact support consists of infinitely many points and is contained in the real line. By Δ we denote the smallest interval which contains the support of s. We denote this class of measures by $\mathcal{M}(\Delta)$. Let

$$\hat{s}(z) = \int_{\Delta} \frac{ds(x)}{z - x}$$

denote the Cauchy transform of s.

$\hat{s}(z)$ is an analytic function in any domain $\Omega \subset \bar{\mathbb{C}} \setminus \Delta$.

Padé approximants

Definition (Padé approximants)

Fix a non zero number $n \in \mathbb{N}$, *then there exists a polynomials* P_n *and* Q_n *such that*

i) $deg\ P_n \leq n-1,\ deg\ Q_n \leq n \quad Q_n \not\equiv 0$

ii) $Q_n(z)\hat{s}(z) - P_n(z) = \mathcal{O}(1/z^{n+1}), \qquad z \to \infty,$

The unique rational function $\frac{P_n}{Q_n}$ *is called diagonal Padé approximants of* $\hat{s}$.

Markov's theorem

Theorem (A.A. Markov,1895)

If Δ is bounded, we have

$$\lim_{n\to\infty} \frac{P_n(z)}{Q_n(z)} = \widehat{s}(z), \quad \mathcal{K} \subset \overline{\mathbb{C}} \setminus \Delta.$$

Markov, A. A. Deux demonstrations de la convergence de certains fractions continues. *Acta Math.* 19 (1895), 93–104.

Let Δ_α and Δ_β be two non intersecting bounded intervals contained in the real line and $\sigma_\alpha \in \mathcal{M}(\Delta_\alpha)$, $\sigma_\beta \in \mathcal{M}(\Delta_\beta)$. With these two measures we define a third one as follows

$$d\langle\sigma_\alpha, \sigma_\beta\rangle(x) = \hat{\sigma}_\beta(x) d\sigma_\alpha(x).$$

Definition (Nikishin system)

Take a collection $\Delta_j, j = 1, \ldots, m$, *of intervals such that*

$$\Delta_j \cap \Delta_{j+1} = \emptyset, \quad 1, \ldots, m-1.$$

Let $(\sigma_1, \ldots, \sigma_m)$ *be a system of measures such that Co*$(supp(\sigma_j)) = \Delta_j$,
$\sigma_j \in \mathcal{M}(\Delta_j)$ $j = 1, \ldots m$.
We say that $(s_{1,1}, \ldots, s_{1,m}) = \mathcal{N}(\sigma_1, \ldots, \sigma_m)$, *where*

$$s_{1,1} = \sigma_1, \quad s_{1,2} = \langle \sigma_1, \sigma_2 \rangle, \ldots \quad , s_{1,m} = \langle \sigma_1, \langle \sigma_2, \ldots, \sigma_m \rangle \rangle$$

is the Nikishin system of measures generated by $(\sigma_1, \ldots, \sigma_m)$.

Take $j \leq k$ we denote

$$s_{j,k} = \langle \sigma_j, \sigma_{j+1}, \ldots, \sigma_k \rangle, \quad s_{j,j} = \langle \sigma_j \rangle = \sigma_j, \quad s_{k,j} = \langle \sigma_k, \sigma_{k-1}, \ldots, \sigma_j \rangle$$

Type II Hermite-Padé approximants

Definition (Type II Hermite-Padé approximants)

Let $s(z) = (s_{1,1}, \ldots, s_{1,m})$ *be Nikishin system. Fix a non zero multi-index* $\vec{n} = (n_1, \ldots, n_m) \in \mathbb{N}_+^m$, $|\vec{n}| = n_1 + \ldots + n_m$. *There exists a polynomial* $Q_{\vec{n}}$ *such that*

i) $deg\ Q_{\vec{n}} \leq |\vec{n}| \quad Q_{\vec{n}} \not\equiv 0$

ii) $Q_{\vec{n}}(z)\hat{s}_{1,j}(z) - P_{\vec{n},j}(z) = \mathcal{O}(1/z^{n_j+1}), \qquad z \to \infty, \qquad j = 1, \ldots, m$

for some polynomials $(P_{\vec{n},1}, \ldots P_{\vec{n},m})$. *The vector of rational functions* $(\frac{P_{\vec{n},1}}{Q_{\vec{n}}}, \ldots, \frac{P_{\vec{n},m}}{Q_{\vec{n}}})$ *is called type II Hermite-Padé approximants of* $\hat{s}$ *respect to the multi-índex* $\vec{n}$.

If $m = 1$, $\frac{P_{\vec{n},1}}{Q_{\vec{n}}}$ is the clasical Padé approximants of $\hat{s}_{1,1}$.

Analog of Markov's Theorem for type II

Theorem

Let $s(z) = (s_{1,1}, \ldots, s_{1,m}) = \mathcal{N}(\sigma_1, \ldots, \sigma_m)$ *be a Nikishin system,* $\Lambda \subset \mathbb{Z}_+^m$ *satisfies*

$$n_1 \geq \ldots \geq n_m \geq n_1 - c, \quad j = 1, \ldots, m, \quad \vec{n} \in \Lambda.$$

Then, for $j = 1, \ldots, m$

$$\lim_{|\vec{n}| \to \infty} \frac{P_{\vec{n},j}(z)}{Q_{\vec{n}}(z)} = \widehat{s}_{1,j}(z), \quad \mathcal{K} \subset \overline{\mathbb{C}} \setminus \Delta.$$

Bustamante, J. and López Lagomasino. Hermite–Padé approximation for Nikishin systems of analytic functions. *Mat. Sb.* 183 (1992), 117–138 (Russian); English translation in Russian Acad. Sci. Sb. Math. 77 (1994), 367–384.

Type I Hermite-Padé approximants

Definition (Type I Hermite-Padé approximants)

Let $s(z) = (s_{1,1}, \ldots, s_{1,m})$ be a Nikishin system. Fix a non zero multi-index $\vec{n} = (n_0, \ldots, n_m) \in \mathbb{N}_+^{m+1}$, where $n_0 - 1 \geq n_j = 1, \ldots m$. There exist polynomials $a_{\vec{n},0}, a_{\vec{n},1}, \ldots, a_{\vec{n},m}$, not all identically equal to zero, such that

i) $\deg a_{\vec{n},j} \leq n_j - 1, j = 0, \ldots, m$

ii) $\sum_{j=1}^{m} a_{\vec{n},j}(z)\hat{s}_{1,j}(z) + a_{\vec{n},0}(z) = \mathcal{O}(1/z^{|\vec{n}|-n_0}), z \to \infty,$

The vector of polynomials $a_{\vec{n},0}, a_{\vec{n},1}, \ldots, a_{\vec{n},m}$ is called type I Hermite-Padé polynomials of $\hat{s}$ respect to the multi-índex $\vec{n}$.

If $m = 1$, $\frac{a_{\vec{n},0}}{a_{\vec{n},1}}$ is the clasical Padé approximants of $\hat{s}_{1,1}$.

Analog of Markov's Theorem for type I

Theorem

Consider a sequence of multi-indices $\vec{n} \in \Lambda$. *Let* $(s_{1,1}, \dots, s_{1,m}) = \mathcal{N}(\sigma_1, \dots, \sigma_m)$ *be a Nikishin system then*

$$\frac{a_{\vec{n},j}}{a_{\vec{n},m}}(z) \rightrightarrows (-1)^{m-j} \hat{s}_{m,j+1}(z) \quad , |\vec{n}| \to \infty \text{ on } \mathbb{C} \setminus \Delta_m \quad j = 0, \dots m-1$$

$$s_{k,j} = \langle \sigma_k, \sigma_{k-1}, \dots, \sigma_j \rangle \qquad (s_{m,1}, \dots, s_{m,m}) = \mathcal{N}(\sigma_m, \dots, \sigma_1)$$

G. López Lagomasino and S. Medina Peralta. On the convergence of type I Hermite-Padé approximants. *Advances in Math. 273 (2015), 124-148. arXiv: 1307.0213v1.*

Analog of Markov's Theorem for type I

Theorem

Consider a sequence of multi-indices $\vec{n} \in \Lambda$. *Let* $(s_{1,1}, \dots, s_{1,m}) = \mathcal{N}(\sigma_1, \dots, \sigma_m)$ *be a Nikishin system then*

$$\frac{a_{\vec{n},j}}{a_{\vec{n},m}}(z) \rightrightarrows (-1)^{m-j} \hat{s}_{m,j+1}(z) \quad , |\vec{n}| \to \infty \text{ on } \mathbb{C} \setminus \Delta_m \quad j = 0, \dots m-1$$

$$s_{k,j} = \langle \sigma_k, \sigma_{k-1}, \dots, \sigma_j \rangle \qquad (s_{m,1}, \dots, s_{m,m}) = \mathcal{N}(\sigma_m, \dots, \sigma_1)$$

G. López Lagomasino and S. Medina Peralta. On the convergence of type I Hermite-Padé approximants. *Advances in Math. 273 (2015), 124-148. arXiv: 1307.0213v1.*

Extensions of Markov's theorem

Uniform Convergence	$\hat{\mu}$		
Padé approximants	✓		

Extensions of Markov's theorem

Uniform Convergence	$\hat{\mu}$	$\hat{\mu} + r$ (Real case)	$\hat{\mu}+r$ (Complex case)
Padé approximants	✓		

Extensions of Markov's theorem

Uniform Convergence	$\hat{\mu}$	$\hat{\mu}+r$ (Real case)	$\hat{\mu}+r$ (Complex case)
Padé approximants	✓	✓	✓

A.A. Gonchar. On the convergence of Padé approximants for some classes of meromorphic functions. *Mat. Sb.* **97(139)** (1975), 607-629.; English transl. in Math. USRR sb 26(1975).

E.A. Rakhmanov. On the convergence of diagonal Padé approximants. *Matem Sb.* **104** (1977), 271–291 (Russian); English transl. in Math. USSR Sb. **33** (1977), 243–260.

Extensions of Markov's theorem

Uniform Convergence	Nikishin system	Real case	Complex case
Type II Hermite-Padé approximants	✓		
Type I Hermite-Padé approximants	✓		

Extensions of Markov's theorem

Uniform Convergence	Nikishin system	Real case	Complex case
Type II Hermite-Padé approximants	✓		
Type I Hermite-Padé approximants	✓	✓	

G. López Lagomasino and S. Medina Peralta. On the convergence of type I Hermite-Pade approximants for rational perturbations of a Nikishin system. *Journal of Computational and Applied Mathematics (2015), http://dx.doi.org/10.1016/j.cam.2015.01.010.*

Extensions of Markov's theorem

Uniform Convergence	Nikishin system	Real case	Complex case
Type II Hermite-Padé approximants	✓	✓	
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U. Fidalgo Prieto, G. López Lagomasino and S. Medina Peralta. Hermite-Padé approximation for certain systems of meromorphic functions. *(Accepted Matematicheskii Sbornik) arXiv: 1310.7010v1.*

Type I Hermite-Padé approximants

Let $(r_1 = \frac{v_1}{t_1}, \ldots, r_m = \frac{v_m}{t_m})$ be real rational functions where the poles of r_j lie in $\mathbb{C} \setminus (\Delta_1 \cup \Delta_m)$ and $deg\ t_j = d_j$; $deg\ v_j < d_j$, for all $j = 1, \ldots, m$.

Definition (Type I Hermite-Padé approximants)

Let $s(z) = (s_{1,1}, \ldots, s_{1,m})$ *be a Nikishin system. Fix a non zero multi-index* $\vec{n} = (n_0, \ldots, n_m) \in \mathbb{N}_+^{m+1}$, *where* $n_0 - 1 \geq n_j = 1, \ldots m$. *There exist polynomials* $a_{\vec{n},0}, a_{\vec{n},1}, \ldots, a_{\vec{n},m}$, *not all identically equal to zero, such that*

i) $\deg a_{\vec{n},j} \leq n_j - 1, j = 0, \ldots, m$

ii) $\sum_{j=1}^{m} a_{\vec{n},j}(z)(\hat{s}_{1,j}(z) + r_j(z)) + a_{\vec{n},0}(z) = \mathcal{O}(1/z^{|\vec{n}|-n_0}), z \to \infty,$

The vector of polynomials $a_{\vec{n},0}, a_{\vec{n},1}, \ldots, a_{\vec{n},m}$ *is called type I Hermite-Padé polynomials of* $\hat{s}_*$ *respect to the multi-índex* $\vec{n}$.

Type I Markov's Theorem for meromorphics functions

Theorem

Consider a sequence of multi-indices $\vec{n} \in \Lambda$. Let $(s_{1,1}, \ldots, s_{1,m}) = \mathcal{N}(\sigma_1, \ldots, \sigma_m)$ be a Nikishin system then

$$\frac{a_{\vec{n},j}}{a_{\vec{n},m}}(z) \rightrightarrows (-1)^{m-j} \hat{s}_{m,j+1}(z) \quad , |\vec{n}| \to \infty \text{ on } \mathbb{C} \setminus \Delta_m \quad j = 1, \ldots m-1$$

and

$$\lim_{\mathbf{n} \in \Lambda} \frac{a_{\mathbf{n},0}}{a_{\mathbf{n},m}} = (-1)^m \widehat{s}_{m,1} - \sum_{j=1}^{m-1} (-1)^{m-j} r_j \widehat{s}_{m,j+1} + r_m.$$

Notice that the rational fractions $(r_1, \ldots, r_m)$ do not play any role in the expression of the limit of $(\frac{a_{\mathbf{n},1}}{a_{\mathbf{n},m}}, \ldots, \frac{a_{\mathbf{n},m-1}}{a_{\mathbf{n},m}})$. On the other hand, all the information of $(r_1, \ldots, r_m)$ is contained in the expression of the limit of $\frac{a_{\mathbf{n},0}}{a_{\mathbf{n},m}}$.

Type II Hermite-Padé approximants

Let $(r_1 = \frac{v_1}{t_1}, \ldots, r_m = \frac{v_m}{t_m})$ be real rational functions where the poles of r_j lie in $\mathbb{C} \setminus (\Delta_1 \cup \Delta_m)$ and $deg\ t_j = d_j$; $deg\ v_j < d_j$, for all $j = 1, \ldots, m$.

Definition (Type II Hermite-Padé approximants)

Let $s(z) = (s_{1,1}, \ldots, s_{1,m})$ *be Nikishin system. Fix a non zero multi-index* $\vec{n} = (n_1, \ldots, n_m) \in \mathbb{N}_+^m$, $|\vec{n}| = n_1 + \ldots + n_m$. *There exists a polynomial* $Q_{\vec{n}}$ *such that*

i) $deg\ Q_{\vec{n}} \leq |\vec{n}| \quad Q_{\vec{n}} \not\equiv 0$

ii) $Q_{\vec{n}}(z)(\hat{s}_{1,j}(z) + r_j) - P_{\vec{n},j}(z) = \mathcal{O}(1/z^{n_j+1}), \qquad z \to \infty, \qquad j = 1, \ldots, m$

for some polynomials $(P_{\vec{n},1}, \ldots P_{\vec{n},m})$. *The vector of rational functions* $(\frac{P_{\vec{n},1}}{Q_{\vec{n}}}, \ldots, \frac{P_{\vec{n},m}}{Q_{\vec{n}}})$ *is called type II Hermite-Padé approximants of* $\hat{s}_*$ *respect to the multi-índex* $\vec{n}$.

Type II Markov's Theorem for meromorphics functions

Theorem

Fix a compact $K \subset \mathbb{C} \setminus \Delta_1$. Assume that $r_1, r_2, \ldots r_m$ have no common finite poles, and all of them lie in $\mathbb{C} \setminus (\Delta_1 \cup \Delta_m)$ then

$$\frac{P_{\vec{n},j}}{Q_{\vec{n}}} \rightrightarrows \hat{s}_j + r_j, |n| \to \infty \text{ on } K \subset \Omega'$$

where $\Omega' = \mathbb{C} \setminus (\Delta_1 \cup \{z : \exists j = 1, \ldots, m : (t_j(z) = 0)\})$

Outline

1 Basics and recents results about convergence of Hermite-Padé approximants

2 Uniform convergence of Mixed type Hermite-Padé approximants

3 Future works

Mixed type Hermite-Padé problem

Definition

Let $n \geq 1$. Given two Nikishin system $(s_{1,1}, s_{1,2}) = \mathcal{N}(\sigma_1, \sigma_2)$, and $(s_{2,2}, s_{2,1}) = \mathcal{N}(\sigma_2, \sigma_1)$. We seek for polynomials $(a_{n,0}, a_{n,1}, a_{n,2})$, $\deg a_{n,0} \leq n-1$,$\deg a_{n,1} \leq n-1$ and $\deg a_{n,2} \leq n$ wich satisfy a mixed type Hermite-Padé approximations conditions as $z \to \infty$

$$a_{n,2}(z)\hat{s}_{2,2}(z) - a_{n,1}(z) = \mathcal{O}(1/z) \qquad (1)$$

$$a_{n,0}(z) - a_{n,1}\hat{s}_{1,1}(z) + a_{n,2}(z)\hat{s}_{1,2}(z) = \mathcal{O}(1/z^{n+1}) \qquad (3)$$

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Hans Lundmark and Jacek Szmigielski. Degasperis-Procesi peakons and the discrete cubic string. *International Mathematics Research Papers, Volume 2005, Issue 2, pp. 53-116. 64 pages. IMRP, Volume 2005, Issue 2.*

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$$a_{n,2}(z)\hat{s}_{2,2}(z) - a_{n,1}(z) = \mathcal{O}(1/z) \quad (1)$$

$$a_{n,2}(z)\hat{s}_{2,1}(z) - a_{n,0}(z) = \mathcal{O}(1/z) \quad (2)$$

$$a_{n,0}(z) - a_{n,1}\hat{s}_{1,1}(z) + a_{n,2}(z)\hat{s}_{1,2}(z) = \mathcal{O}(1/z^{n+1}) \quad (3)$$

Hans Lundmark and Jacek Szmigielski. Degasperis-Procesi peakons and the discrete cubic string. *International Mathematics Research Papers, Volume 2005, Issue 2, pp. 53-116. 64 pages. IMRP, Volume 2005, Issue 2.*

Mixed type Markov's Theorem

Theorem

Consider a sequence of vector polynomials $(a_{n,0}, a_{n,1}, a_{n,2})$ *that satisfies the previous mixed type approximation problems then*

$$\frac{a_{n,1}}{a_{n,2}}(z) \rightrightarrows \hat{s}_{2,2}(z) \quad , n \to \infty \text{ on } \mathbb{C} \setminus \Delta_2$$

$$\frac{a_{n,0}}{a_{n,2}}(z) \rightrightarrows \hat{s}_{2,1}(z) \quad , n \to \infty \text{ on } \mathbb{C} \setminus \Delta_2$$

An idea of the proof.

- $a_{n,0}(z) - a_{n,1}\hat{s}_{1,1}(z) + a_{n,2}(z)\hat{s}_{1,2}(z) = \mathcal{O}(1/z^{n+1})$
- $\int_{\Delta_1} x^{\nu}(a_{n,2}(x)\hat{s}_{2,2}(x) - a_{n,1}(x))ds_{1,1}(x) = 0,\ \nu = 0, n-1$
- $(a_{n,2}(x)\hat{s}_{2,2}(x) - a_{n,1}(x))$ has at least n simples zeros on Δ_1
- $\frac{a_{n,2}(z)\hat{s}_{2,2}(z) - a_{n,1}(z)}{w_n(z)} = \mathcal{O}(1/z^{n+1}), z \to \infty,$
- $a_{n,1}/a_{n,2}$ is a multipoint Padé approximant of $\hat{s}_{2,2}$

  Bustamante, J. and López Lagomasino. Hermite–Padé approximation for Nikishin systems of analytic functions. *Mat. Sb.* 183 (1992), 117–138 (Russian); English translation in Russian Acad. Sci. Sb. Math. 77 (1994), 367–384.
- Using some properties of Nikishin system and the previous ideas we obtain the convergence of $a_{n,0}/a_{n,2}$ to $\hat{s}_{2,1}$

□

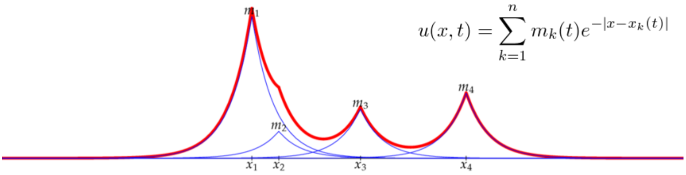
Degasperis-Procesi Peakons and the Discrete Cubic String

Degasperis-Procesi equations

$$u_t - u_{xxt} + 4uu_x = 3u_x u_{xx} + uu_{xxx}, (x,t) \in \mathbb{R}^2$$

This equations admit (in a weak sense) a type of nonsmooth solutions called multipeakons (peakon = peaked soliton). These take the form of a train of peak-shaped interacting waves,

$$u(x,t) = \sum_{i=1}^{\infty} m_i(t) e^{-|x-x_i(t)|}$$



Discrete cubic string problem

Discrete cubic string problem

Given a function $g(y) \geq 0$, determine the eigenvalues z such that nontrivial eigenfunctions $\phi(y)$ exist, satisfiying

$$-\phi_{yyy}(y) = zg(y)\phi(y), \quad \text{for } y \in (-1, 1)$$

$$\phi(-1) = \phi_y(-1) = 0, \quad \phi(1) = 0.$$

Mixed type Hermite-Padé problem

Definition

Let $n \geq 1$. Given two Nikishin system $(s_{1,1}, s_{1,2}) = \mathcal{N}(\sigma_1, \sigma_2)$, and $(s_{2,2}, s_{2,1}) = \mathcal{N}(\sigma_2, \sigma_1)$. We seek for polynomials $(a_{n,0}, a_{n,1}, a_{n,2})$, $\deg a_{n,0} \leq n-1$,$\deg a_{n,1} \leq n-1$ and $\deg a_{n,2} \leq n$ wich satisfy a mixed type Hermite-Padé approximations conditions as $z \to \infty$

$$a_{n,2}(z)\hat{s}_{2,2}(z) - a_{n,1}(z) = \mathcal{O}(1/z)$$
$$a_{n,2}(z)\hat{s}_{2,1}(z) - a_{n,0}(z) = \mathcal{O}(1/z)$$
$$a_{n,0}(z) - a_{n,1}\hat{s}_{1,1}(z) + a_{n,2}(z)\hat{s}_{1,2}(z) = \mathcal{O}(1/z^{n+1})$$

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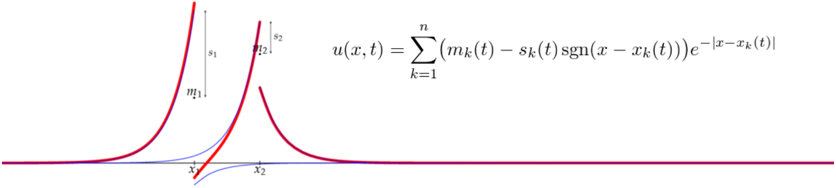
The mixed type Hermite Padé approximation problem can be extended in the sense of consider a Nikishin system of m measures. Moreover, the results about uniform convergence can be obtained consider not only the case of discrete measures.

Problem

How this extension of the approximation problems and the results of the uniform convergence when the measures are not discrete are related with the inverse spectral problem for the string and the Degasperis-Procesi equation?

For the Degasperis-Procesi equation a more general case deals with peakons which move to right and atipeakons which to left. In this situation the approximation problem which arises from the Degasperis- Procesi equation is with respect to a rational pertubation of the Nikishin system.



$$u(x,t) = \sum_{k=1}^{n} \big(m_k(t) - s_k(t)\,\mathrm{sgn}(x - x_k(t))\big) e^{-|x - x_k(t)|}$$

Problem

Can we prove an explicit peakon-antipeakon formula in this more general case?

THANK YOU!!!